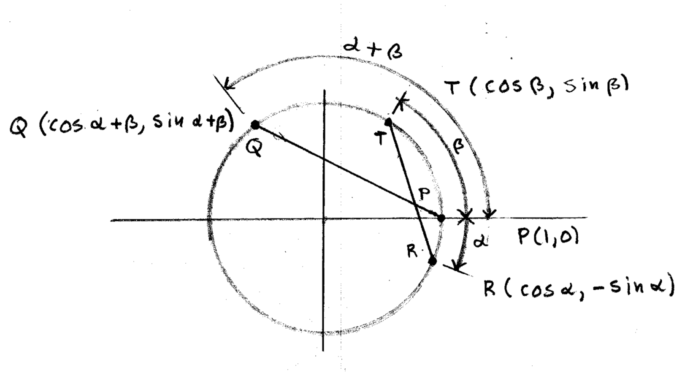
[09-11-14-RT-11]

Proof of addition theorem



Assume as in the figure above that the sum of an arc of length α and an arc of length β is an arc of length $\alpha + \beta$.

Since $\overset{\text{arc}}{\text{PQ}} = \overset{\text{arc}}{\text{RT}}$, the lengths chords PQ and RT are equal.

PQ^2

$= \;[\cos(\alpha+\beta) - 1]^2 + [\sin(\alpha+\beta) - 0]^2$

$= \;\cos^2(\alpha+\beta) - 2\cos(\alpha+\beta) + 1 + \sin^2(\alpha+\beta)$

$= \;2 - 2\cos(\alpha+\beta)$

RT^2

$= \;[\cos\beta - \cos\alpha]^2 + [\sin\beta + \sin\alpha]^2$

$= \;\cos^2\beta - 2\cos\alpha\cos\beta + \cos^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \sin^2\alpha$

$= \;2 - 2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta$

Since PQ = RT

PQ = RT

$\Longleftrightarrow 2 - 2\cos(\alpha+\beta) = 2 - 2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta$

$\Longleftrightarrow -2\cos(\alpha+\beta) = -2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta$

$\Longleftrightarrow \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

Therefore,

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

□